The Application of Mathematical Methods to the Determination of Transport Flows

Primjena matematičkih metoda kod određivanja prometnih tokova

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Summary

This article deals with the basic mathematical methods applied to the determination of the size of traffic flows in an urban area transport network. The methods were applied to the design of a new system of public bus transport in the city of České Budějovice. This article represents the partial outcome of the research carried out by the Institute of Technology and Businesses in České Budějovice.

KEY WORDS

operational analysis methods
solving optimization problems
public bustransport
decision making
Detroit model

Sažetak

U ovom radu govori se o osnovnim matematičkim metodama koje se primjenjuju kod određivanja veličine prometnih tokova u urbanoj prometnoj mreži. Metode su primijenjene na dizajn novog sustava javnog autobusnog prijevoza u gradu Česke Budejovice. U radu je izložen dio rezultata istraživanja koje je proveo Institut tehnologije i poslovanja u Českim Budejovicama.

KLJUČNE RIJEČI

metode operacijske analize
rješavanje problema optimizacije
javni autobusni prijevoz
donošenje odluka
model Detroit

INTRODUCTION

Transport planning and operational transport management is a field where the application of simulation models based on mathematical methods is the first step in the determination of an efficient solution. Mathematics is applied in transport system modelling as a basic tool, whether it deals with a network of airlines or a network of railway routes. This article outlines the application of basic mathematical methods particularly in the creation and optimization of a network of municipal public transport lines. The optimization or rationalization of already existing transport systems is actually more usual under the present conditions.

We can distinguish between strategic and operational planning in the public transport sphere. In the first phase of strategic planning, we can collect the basic data, e.g. on passenger intensities (O/D matrix) and set the strategic goals of transport service in relation to the city and its inhabitants. Numerous statistical and stochastic methods are applied in this phase, but also basic mathematical methods are applied as well, e. g. particularly for the determination of transport flows within a system. In the next phase of operational planning, the obtained data are processed to the best possible effectiveness of the transport system with the stress on cost minimization. The proposal of line routes within the given territory (that includes the calculation of vehicle turns on these lines and other processes linked to the optimization) can be included into this phase of the transport system optimization. Mathematical methods have to necessarily be integrated into these processes. The determination of the optimum number of vehicles and planning their turns is a typical case where the basic methods of operational research are applied, e. g. the "Travelling Salesman Problem" [1].

Nowadays, when there are still new demands arising in transport and transport systems are on the increase, mathematics is even more necessary and important for decision making problems in transport planning. Operational research and optimizing mathematical methods have a long history in air transport; for example, decision making problems are supported by operational analysis methods and have already been developing for 30 years. Airlines use optimization techniques for daily, weekly, and monthly planning such as fleet assignment, crew scheduling, and crew fostering [1].

Fleet assignment in forwarding companies may similarly be optimized or linear programming methods can be used for the determination of traffic flow utilization on roads and motorways, for thecreation of a future development model to predict congestion occurrence. The application opportunities are numerous. So let us focus on the application of certain mathematical methods to the determination of the size of the transport flows between individual areas within a municipal public transport system.

THE GENERATION OF TRANSPORT FLOWS BETWEEN THE AREAS OF A TRANSPORT SYSTEM

The complex technique of transport system modelling only developed in the times when problems requiring complex and coordinated solutions were growing on one side and sufficiently powerful computing technology that enabled such solutions was available on the other side. The whole process of transport system modelling can be split into four parts (the traditional four-step transport model)[2]:

1. The generation of transportation needs (volumes of origin and destination transport).
2. The distribution of transportation relations (directions of transport flows).
3. The classification of transportation relations according to the means of transport used (division of transportation labour).
4. The utilization assignment to routes and transportation network sections.

Network transport research has to be done in order to obtain the intensities of passengers within a public transport system. The territory where the transport services within the transport network are provided $S = (V, H)$, has to be divided into areas (zones). A better solution is to divide the territory to as many zones as there are nods $v \in V$ in V. Each zone is thus allocated to a nod. Transport intensities q_{ij} b between two different zones Q_i and Q_j may then be determined right from the research performed. Unless inter-zonal transport links are known, it is also possible to take the transport volume into account q_i getting out of the zone (and thus also getting in) in the calculation and calculate these values, for example by multiple regression analysis in linear form.

The basic formula for calculation and determination of inter-zonal transport links may be considered as follows:

$$f = \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_j \,, \tag{1}$$

where x_{ij} is a variable, which may be for example the distance between the i-th and the j-th zones, the price of transport between the zones or the travel time between the zones, n is the number of considered zones. The following conditions however have to be met at the same time:

$$\sum_{i=1}^{n} q_{ij} = q_i \; i = 1,\ldots..n \tag{2}$$

$$\sum_{i=1}^{n} q_{ij} = \overline{q_j} \; j = 1,\ldots..n \tag{3}$$

TRANSPORTATION FORECASTING

The application of the four-step transportation model to transport forecasting requires transport research, evaluation of the exact values of the transport process, the derivation of the dependences valid for the present conditions and the prognosis of the expected transport flows may be calculated based upon these data. However, this includes the integration of various socio-demographic, developmental and other data and the prospective transport volumes may be based on them.

THE CALCUATION OF THE PROSPECTIVE TRANSPORT VOLUMES

Multifactor analysis procedures, namely multiple regression analysis, were commonly used for the determination of the prospective transport volumes of the individual areas. The transport volume was calculated as a variable in dependence on various socio-demographic data of the area as relevant independently variable structural quantities:

$$D_i = a + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n \,, \tag{4}$$

where

D_i - is a dependent variable (transport volume of the i-th area)

a - regression equation constant

$b_1, b_2, \ldots, b_n$ - partial regression coefficients

$X_1, X_2, \ldots, X_n$ - independently variable structural quantities with decisive influence on transport volume

Double regression analysis was used for the calculation of the prospective transport volumes as the simplest method, in which numbers of inhabitants and labour opportunities were used as structural quantities. Linear regression analysis was examined in Slovakian Bratislava within the Czechoslovak state research task of P-13 order with the introduction of eight independently variable quantities that were available in the detailed territorial division: the active and passive inhabitants, labour opportunities in sectors I and II, labour opportunities in sector III divided into active and passive, the numbers of places at primary schools, secondary and tertiary schools and other equipment of the area in m^2 of usable area [3].

The calculations were performed for models with fixed input of structural quantities in a certain order, with the classification of the individual structural quantities according to the link strength always for equations with an absolute term (with regression equation constant) and for equations with a limiting condition without an absolute term.

Although the results were very good, there were still some doubts whether linear regression analysis is the right procedure for the determination of the transport volumes. There is a serious reservation that it does not provide a view of establishment and expiry of the transport links from the transport engineering point of view. The regression analysis was gradually replaced in calculation of transport volumes by different methods, particularly the Specific Momentum Method, which however requires more detailed calculation inputs.

TRANSPORT FLOW ROUTING

After the phase of the determination of the transport volumes between the individual zones within a transport network, the determination of directivity of these transport flows may follow. This means the determination of the size of each transport link D_{ij} between two zones *i* for starting trips and *j* for ending trips and thus create a complete matrix of links for all *n* districts the territory is divided into.

The calculation of the prospective directing of the transport flows has also developed from the simplest one-factor procedures, via the average growth factor to more complex analogical and synthetic procedures.

The requirement for the sums of the volumes of the source and destination transport (that are to be equal and to equal the total transport volume of the monitored territory) is the basic condition for all these procedures. There are further marginal conditions, mainly the requirements for the sum of all trips from area *i* to the other areas *j* to equal the volume of the source transport of area *i* and for the sum of all trips to area *j* from all areas to equal the volume of destination transport of area *j*. These conditions cannot usually be met for the first time; so the calculation is repeated with gradual approximation (iteration) and after the achievement of certain accuracy, the procedure is finished.

Analogical procedures determine the prospective transport links by the analogy with the present situation, while the simplest of them only takes account of the basic requirements (the destination and source transport volumes, the source-destination distance etc.). The oldest one (the uniform growth factor procedure) assumes that all inter-zonal relations grow uniformly, i.e. that the growth factor is identical for the whole monitored territory and is defined by the quotient of the prospective and the present transport volumes. In the average growth factor procedure, the prospective transport link is determined by the product of the present link and the arithmetic average of the growth factor of source transport *i* and destination transport *j* of the area.

More complex analogical procedures, like the Detroit and Fratar Models already take account of not only the pair of monitored areas, but also the mutual influence of the other areas of the territory. The Detroit Model is based on the assumption that the prospective transport link is directly proportional to the present link and the growth factor of both the monitored areas and is indirectly proportional to the growth factor of the whole city [2]:

$$D_{ij}^{v} = D_{ij}^{s} * \frac{K_i^Z * K_j^C}{K}, \tag{5}$$

where

D_{ij}^{v} - prospective transport link,

D_{ij}^{s} - present transport link,

K_i^Z - growth factor of source transport in area*i*,

K_j^C - growth factor of destination transport in area *j*,

$K = \frac{D^{v}{}_{loc.district}}{D^{s}{}_{loc.district}}$ - whole city growth factor

Synthetic procedures seek various ways of expressing the individual factors for the future, as they significantly influence the size of the prospective transport link D_{ij}. The gravity model in various modifications is particularly used in transport engineering.

THE USE OF THE MATHEMATICAL BASIC METHOD TO PROPOSE LINK LINES

Let us now have a look at an example of the application of the basic mathematical methods to the preparation of a transport model. It is namely a transport problem solving algorithm applied to an assignment problem solution. This method has been chosen particularly to show that we are able to save a substantial part of costs, even by means of a principally simple mathematical method, which would have been many times higher if the same situation had been solved by for example the VISEVA, VISUM (Modal Split) model[4].

The transport model designed this way was used within the research also for optimization of public transport routing in České Budějovice. The solution itself was preceded by the transport research on the public transport lines, from which the transport volume data were used.

THE MATHEMATICAL MODEL OF THE ASSIGNMENT PROBLEM

The optimization of the transport links in public transport can be formulated and solved as an assignment in which the individual districts are regarded both as sources and destinations. The rates are sums of the identified source and destination traffic intensities between any two districts. In order to avoid assigning the same two districts, so called prohibitive rates were chosen on the main diagonal (see *table 1*). The criterion of optimality is the maximization of the total number of passengers transported without transfer.

The mathematical model of the problem has the following form [5], [6]:

To maximize:

$$f = \sum_{j=1}^{26}\sum_{i=1}^{26} c_{ij}\, x_{ij} \tag{6}$$

$x_{ij} = 1$, if the i-th transport district is assigned to the j-th transport district

$x_{ij} = 0$, in the opposite case

In case of restrictions:

$$\sum_{j=1}^{26} x_{ij} = 1, \qquad i = 1,2,3 \ldots 26$$
$$\sum_{i=1}^{26} x_{ij} = 1, \qquad j = 1,2,3 \ldots\ldots 26 \tag{7}$$

The classical task assignment is characterized by minimization. However, in our case it is necessary to maximize the traffic flows. In this case,we only need to replace the objective function f with the function [12], [17], [18]:

$$f = \sum_{j=1}^{26}\sum_{i=1}^{26} (-c_{ij}) x_{ij} \tag{8}$$

Table 1 Adjusted Table - The Sum of the Source and Destination Traffic Intensities - prohibitive rates on the main diagonal

Adjusted matrix – sums of source and destination transport intensities – prohibitive rates on the main diagonal

Values obtained by research

Total data for 23rd November (5.30 – 11.30) and 24th November (11.30 – 17.30)

Number	z A	z B	z C	z D	z E	z F	z G	z H	z I	z J	z K	z L	z M	z N	z O	z P	z Q	z R	z S	z T	z U	z V	z W	z X	z Y	ze Z
do A	-1000	75	390	109	20	41	10	50	228	619	2	10	1937	998	149	89	24	207	168	292	167	268	4	298	328	31
do B	120	-1000	16	1	9	2	1	0	10	175	0	2	24	46	1	8	9	9	17	20	0	6	0	7	34	1
do C	1299	46	-1000	175	125	133	26	46	320	351	55	245	219	68	6	26	166	362	365	400	69	212	192	113	48	31
do D	60	2	162	-1000	27	5	0	3	29	106	0	3	6	248	2	11	13	7	22	77	5	25	1	27	11	7
do E	45	7	94	19	-1000	23	0	3	152	88	1	2	249	86	3	13	5	49	31	26	43	9	1	20	2	4
do F	158	1	278	6	16	-1000	2	6	11	41	0	0	173	168	0	11	19	16	7	25	2	52	0	57	13	8
do G	47	3	42	2	21	23	-1000	2	13	40	2	14	3	74	0	2	5	20	11	9	2	17	0	11	7	7
do H	50	3	113	2	11	8	0	-1000	146	72	0	4	146	150	1	7	9	5	64	16	0	7	0	146	1	2
do I	538	11	152	28	260	36	1	158	-1000	269	21	64	308	233	2	83	115	119	202	352	184	53	7	111	12	147
do J	1356	317	692	120	232	57	22	63	289	-1000	0	158	423	644	7	86	146	397	407	322	73	109	105	74	63	55
do K	18	0	38	2	1	0	2	27	33	3	-1000	0	42	131	3	3	53	7	18	23	1	4	0	46	0	3
do L	21	0	318	3	20	2	2	29	40	75	0	-1000	68	75	0	3	52	2	29	6	1	10	0	4	0	0
do M	2165	27	145	38	266	160	1	140	282	276	44	85	-1000	245	10	25	205	162	355	964	275	286	2	156	69	27
do N	1505	23	162	288	285	150	60	96	395	487	43	30	272	-1000	220	135	175	145	411	388	249	386	68	123	106	469
do O	147	1	12	4	14	17	0	5	9	41	0	6	41	171	-1000	13	0	16	15	52	36	50	0	53	2	8
do P	98	7	7	11	81	12	0	7	136	48	0	0	3	178	0	-1000	58	23	10	53	3	17	0	27	3	44
do Q	150	6	37	17	7	5	1	39	151	49	6	3	115	151	1	75	-1000	36	56	22	7	15	1	88	4	44
do R	185	131	256	15	43	15	3	6	80	307	0	1	167	183	12	21	27	-1000	145	38	49	22	0	24	33	7
do S	173	13	496	4	8	10	4	5	141	439	0	4	228	450	3	16	19	140	-1000	91	10	77	11	78	16	14
do T	484	19	248	92	59	28	0	34	216	305	9	11	847	314	14	39	10	84	115	-1000	52	138	4	222	92	34
do U	236	4	79	12	189	13	0	10	109	86	0	4	267	111	22	7	18	50	45	95	-1000	35	0	97	0	7
do V	139	3	151	29	0	37	5	5	40	30	2	2	167	208	16	24	19	13	38	51	0	-1000	0	37	3	11
do W	5	0	195	0	8	3	1	0	1	48	0	0	2	56	1	4	4	3	26	4	0	7	-1000	6	0	4
do X	204	8	67	27	1	20	0	53	82	93	19	19	241	147	27	39	5	22	110	177	23	60	4	-1000	3	57
do Y	425	47	49	80	20	29	0	2	26	151	0	2	315	102	5	16	5	81	51	95	10	85	0	7	-1000	7
do Z	31	0	2	15	17	10	1	8	198	21	0	2	11	734	0	59	62	7	20	10	3	10	1	59	4	-1000

Source: authors

THE SOLUTION OF THE ASSIGNMENT PROBLEM AND INTERPRETATION OF THE RESULTS

The assignment problem is usually solved by the so called Hungarian Method [4], [7]. Due to the software options,the considered assignment problem was solved as a transportation problem using the Dumkosa program (an add-in XLA, created by the Department of Operational and Systems Analysis at PEF ČZU in Prague). Following the theory of linear programming, the solution of an assignment problem obtained by the methods for solving the transport problem is greatly degenerated. Completing the number of occupied boxes to the number required for a non-degenerated solution is usually done by using a negligibly small amount of EPS. The symbol ALT in some fields means that by filling this field we would obtain the equivalent optimal solution with the same value of the objective function (samples for first and fourth grade directional transport flows and are indicated in *tables 2 and 3*). Using the Dumkosa Program, first grade transport flows were obtained; the strongest links are between the hubs (see *table 2*). However, the algorithm suggests such connections so as the value of the objective function (the sum of all transported passengers without transfer) is at its maximum at any given moment [5], [6].

The procedure was then repeated four times. The procedure was always the same, but further prohibitive rates were put into the places of the assigning rates. As a result, four transport flows were obtained (see *tables 2 - 5*) which are marked for the sake of simplicity. It is necessary to point out that the first grade is the most significant one. It is also very important to realize that the obtained grades of the transport flows are not comparable given the importance of the individual districts. For this reason, it is necessary to consider every transport grade as significant in the case of a large district (estate), whereas it is possible to consider only the first and possibly the second grade transport flows in the case of a weak transport district (e. e. Havlickova Kolonie). The proposed allocation of the districts represents the best directlinks according to the selected mathematical model [8].

A situation where individual transport flows come together or coincide is very crucial in this context. A typical case of this occurs in districts A and S (*table 1*).

The proposed direct transport links represent the best direct link lines of public transport in České Budějovice while respecting the criterion of optimality where by the maximum number of passengers travel direct and without transferring. For those transferring, it is proposed to minimize waiting times based on the transport theory: limiting parallel lines following the same stretches to different destinations. Furthermore,suitable transfer terminals can be read out of the model (*following tables 2 and 3*).

According to the above-mentioned mathematical algorithm 4 transport flow grades were identified (samples for first and fourth grade directional transport flows and are indicated in *tables 2 and 3*) [8].

Table 2 Optimal Solution of the Transport Model - First Grade Directional Transport Flows

Optimal solution of a transport model - Directional transport streams of first degree outside of main diagonal

Optimal value of purpose function is 16402

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	EPS	0	0	0	0	0	0	EPS	0	0	0	EPS	0	0	EPS	0	0	0	0	1	0	0	0	1
C	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1	0	1
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1	0	0	0	0	1
G	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	EPS	0	0	0	0	EPS	1
J	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	1	0	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	EPS	0	0	0	0	0	0	0	0	0	1
L	0	0	EPS	0	0	0	1	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
M	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
O	0	0	0	ALT-	0	0	0	0	0	0	1	EPS	0	0	0	0	0	0	0	0	EPS	0	0	EPS	0	0	1
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
Q	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
R	0	EPS	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
T	0	0	0	0	0	0	0	0	1	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	EPS	0	0	1
U	0	0	0	0	1	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
V	0	0	0	0	0	1	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
W	0	1	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	1	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	1
Y	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Source: authors

Table 3 The Optimal Solution of the Transport Model – Fourth Grade Directional Transport Steams

Optimal solution of a transport model - Directional transport streams of fourth degree outside of main diagonal

Optimal value of purpose function is 8856

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
B	0	0	0	0	0	0	1	0	0	0	0	EPS	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	1
C	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	1
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	1	0	0	0	0	1
E	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	EPS	EPS	0	1
G	0	ALT-	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
H	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	1
J	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
L	0	1	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	1
M	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	EPS	0	0	0	0	1
N	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1
O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
P	0	EPS	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
S	0	0	0	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	1	0	0	0	0	0	EPS	0	0	1
T	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
U	0	0	0	0	0	EPS	0	0	EPS	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
V	0	0	0	1	0	0	0	0	0	0	0	0	0	EPS	0	0	0	0	0	EPS	0	0	0	0	0	0	1
W	0	0	0	0	0	0	0	0	0	0	EPS	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	EPS	1
Y	0	0	0	0	0	EPS	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

Source: authors

THE COMPREHENSIVE PROPOSAL FOR THE ROUTING OF THE LINKS OF URBAN AND SUBURBAN TRANSPORT LINES

Based on the results of the mathematical model the distribution of direct transport links was determined. It was necessary to move from the stage when the individual flows are proposed using a mathematical model to the stage where from a number of proposed options such options must be selected which follow on from the transport significance of a district (stated by the decision maker), from the stretch intensity between districts and from basic transport principles (especially the classical theory of transport).

This procedure also requires the use of common sense, experience and the knowledge of the history of the transport system. It is therefore necessary to also apply the personal approach of a decision maker. It should be noted that as part of the decision making process.It is also necessary to take into account the system of trolleybus transport, especially the existing trolleybus network.

The methodical solution applied in the research of the transport model proposed with the support of mathematical methods represents a modern approach to designing the routes of the lines in České Budějovice and it can be applied to any city public transport system. The implemented changes represent a significant positive contribution to the overall public transport system. Compared to 2000, the number of passengers carried has increased by at least 30% (on the Máj – Nádraží route). The system is gradually becoming more "user friendly" due to the introduction of the routes that passengers use [8],[9].

Another benefit is the expansion of the trolleybus network in České Budějovice. A new trolleybus line to České Vrbné was completed in 2007.

From the point of view of scientific knowledge, this study is an example of creative research applied to a purely practical problem with the use of exact mathematical methods. Furthermore, this research has confirmed that it is necessary to know its substance as well as modern methods for its solution for a successful solution to any problem [10]-[13].

CONCLUSION

The theoretical part of this article outlines the mathematical methods that can be used for the determination of the transport links between individual areas within a territory, or actually the transport flows and their directionality. The practical part of this article contains the determination of the transport flows in the territory of České Budějovice from the input data by means of basic mathematic methods. The new municipal public transport lines were also proposed and the existing ones were optimized upon the determined transport flows.

The benefits cited in the previous chapter are quite significant and it is undeniable that research brings not only new insight into some mathematical methods, but also into the possible marketing or economic aspects when designing or modifying transport models, and not only in public transport sector.

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