

# Optimal Use of Container Ships for Servicing Among Small Ports

## Optimalna uporaba kontejnerskih brodova koji opslužuju male luke

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DOI 10.17818/NM/2018/1.3

UDK 629.56:656.615

Preliminary communication / Prethodno priopćenje  
Paper accepted / Rukopis primljen: 12. 2. 2018.

### Summary

Transportation of containers, in areas which are not interesting for large shipping operators, can pose a problem for ship cargo officer due to limited logistical assistance from ashore. In such cases, all calculations and planning have to be done on board the ship. Transportation between secondary ports to other-final ports, e.g. for servicing ports in island region, can be done by independent carriers which operate small container ships without proper logistical assistance. The efficient algorithm for optimal transport of  $N$  contingents of containers, for ship with limited capacity on the route with  $M$  ports, is being considered. The proposed algorithm has acceptable complexity and such optimization tool can be used efficiently in limited shipping surrounding. But crucial condition is well educated cargo officer, to be skilled about such management and planning tool.

### Sažetak

Prijevoz kontejnera, posebno u područjima koja velikim prijevoznicima nisu zanimljiva, može predstavljati problem časniku palube zaduženom za organizaciju tereta zbog ograničene logističke podrške s kopna. U takvim slučajevima svi izračuni i planiranje moraju biti obavljani na brodu. Prijevoz između malih luka prema drugim lukama odredišta, npr. u opsluživanju otočnih luka, može obavljati neovisni prijevoznik koji upravlja malim kontejnerskim brodom bez odgovarajuće logističke podrške. Učinkoviti algoritam za optimalni prijevoz  $N$  vrsta kontejnerskih kontingenata, za brod ograničenog kapaciteta, na liniji s  $M$  luka je predložen. Predloženi algoritam prihvatljive je kompleksnosti i takvo računsko sredstvo može biti uspješno primijenjeno u ograničenom okruženju broda. Jedini ključan uvjet je dobra edukacija časnika zaduženog za teret da uz pomoć ovog alata može uspješno donositi odluke o planu krcanja.

### KEY WORDS

non-linear transportation problem  
multi-destination routing problem  
capacity management of container ships

### KLJUČNE RIJEČI

nelinearni transportni problem  
problem usmjeravanja prometa za  
više odredišta  
upravljanje kapacitetom  
kontejnerskog broda

## 1. INTRODUCTION / Uvod

During last 60 years container ship trade grew significantly, which forced operators to look for more efficient ways for better transportation planning [1, 2]. The main goal of all involved parties is to lower overall transportation costs and to reduce congestion of infrastructure components, by delivering containers to the destination, as soon as possible. Transportation is organized on the principle of "hub and spoke" network modeled on the aircraft industry. These are linear network between ports that meet the needs of „mega“ container ships, starting on one side and ending at the other side of the world. Among them sail large ships called "mother" ships and further distribution between main and secondary ports is done with smaller container ships [3, 4].

For container ships of great size, with set timetable for loading/discharging ports, situation is clear; cargo must be delivered as per plan. Situation is similar for container ships which serves "hub-secondary port" route.

Before operator even starts planning, first task is to find ship with optimal size, draft, maneuvering characteristics and stowage capacity [4, 5]. Potential implications regarding deploying too small or big ship may have adverse effect on venture. For these stages of planning all calculations are done by ship cargo operators which are based ashore, with full logistical and computing assistance.

On the other side, transportation of containers between secondary ports to other-final ports, in this scenario placed on the islands, is done by independent carriers which operate small container ships without proper logistical assistance. Route planning is usually done by ship cargo officer and can pose a great problem.

When there is more than one port involved in transportation planning, important inputs like transshipment costs, fuel price for each route, sailing time, time for loading /discharge containers and the location of each port present main inputs to

determine the overall efficiency of the process [4]. In addition to the above, other conditions have impact on process in whole like an efficient operator network, efficient logistical support and most important acceptable port conditions (acceptable entry time-window, weather conditions, pilotage availability etc.) Therefore, operator can adjust flexibility when set schedule is deranged and to improve plan as required.

To minimize the operational cost of the venture all aspects stated must be considered and integrated into plan. This paper addresses this need by proposing a model that considers the port locations together with the operational cost.

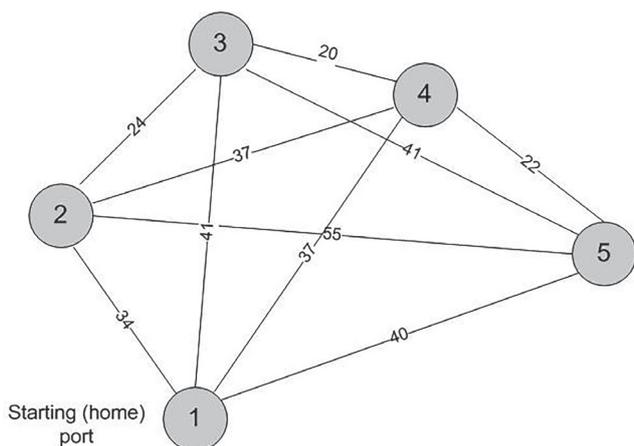


Figure 1 Distances between the ports  
Slika 1. Udaljenosti između luka

The capacity management problem in container shipping is extended to the transportation problem of different contingents (cargo types) transported by one ship on the route with multiple sources (loading ports) and multiple destinations (ports of discharge). Loads of containers (empty and loaded) are waiting to be transported as it is shown on fig 2. The load amounts are given in percentage of total ship capacity. If all contingents have the same transportation cost it is clear that the ship will service all ports and be backed to home port. It will be happened only if total income is positive. But if it is not the case the avoidance of some port could be possible, in order to find optimal transportation cost. Amount of different cargo loads (e.g. container) is in firm correlation because the total capacity of the ship is limited in TEU (Twenty-foot Equivalent Unit). Taking into account the cargo waiting to be transported, we need optimal transportation plan to minimize shipping and loading/unloading expenses, transshipment cost and cost of ship's stay in port (connected with duration of loading process). Also, it can help in definition of ship capacity arrangement or for comparison of ships with different cargo capacity.

The non-linear transportation problem (NTP) with multiple (several) ports of loading (sources) and multiple destinations (sinks) is very hard (NP-hard) problem so it is still the subject of many scientific papers. The similar problem can be solved with different techniques, see [6-12].

In special circumstances the NTP can be seen as Minimum Cost Multiple Commodities Flow Problem (MCMCF); see [13] and [14]. In this paper we applied such network optimization approach. The mathematical model is formulated in section 2. Implemented algorithm is tested on some examples and results are commented in section 3.

In example from fig. 1 we set 5 ports with set distances. For this purpose the container ship is limited with capacity, e.g. of max. 100 TEU, in order to reach maximum cost utilization. All information about potential load can be gathered through market research or from statistics. On diagram from fig.2. we can see eight contingents waiting for transport, half of them are for empty containers (1-2,1-3,1-4-,1-5) and second half are for loaded containers (2-1,3-1,4-1,5-1).

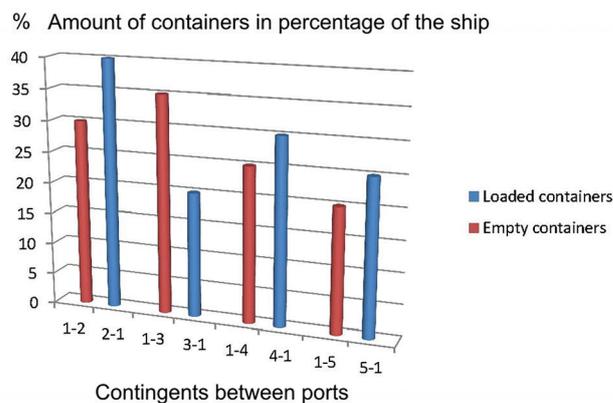


Figure 2 Potential transfer of container contingents between ports given in percentage of the total ship capacity  
Slika 2. Potencijalni tereti (kontingenti) kontejnera koji čekaju na prijevoz izraženi u postotku brodskog kapaciteta

For test-example from fig. 2. we have optimal solution represented on fig. 4 – fig. 6. On fig. 6. it is obvious that we have almost full ship, entering in all ports, with only small amounts of idle space (5 - 15%) on the voyage from 1-2 (10%), 3-4 (5%), 4-5 (15%) and. 5-1 (10%). In that case the transport cost reduction is more important that idle (unused) capacity on board.

## 2. MATHEMATICAL MODEL / Matematički model

Transportation of empty and loaded container contingents is specific problem but it can be seen as similar transport problems [15-18]. Various types of container contingents are differentiated with  $i$  for  $i = 1, 2, \dots, N$ , odd for empty and even for loaded containers, see fig.3 . The ship with defined cargo capacity is shipping from the starting port, servicing number of  $K$  potential ports. The objective is to find a loading and transshipment strategy that minimizes the total cost incurred over the whole voyage route consisting of  $M$  ports on the path ( $M \leq K$ ). Each port on the route can be for loading and for discharging.

The transportation problem can be represented by a flow diagram on fig. 3. The problem can be solved with different techniques, see [3-9] but here we applied network optimization technique as the shortest path problem in the network with multiple sources/destinations. Some ports have limitation on loading capacity and most of them are secondary ports with capacity below the ship's earning capacity. In this paper the contingents are transported from/to home port, but in general, any combination of starting/ending ports can be introduced. On figure 3. the  $i$ -th row of nodes represents the capacity state of  $i$ -th type of contingent after loading in port  $m$ . The links between nodes represent the amount of cargo transported between ports (in percentage of ship capacity).

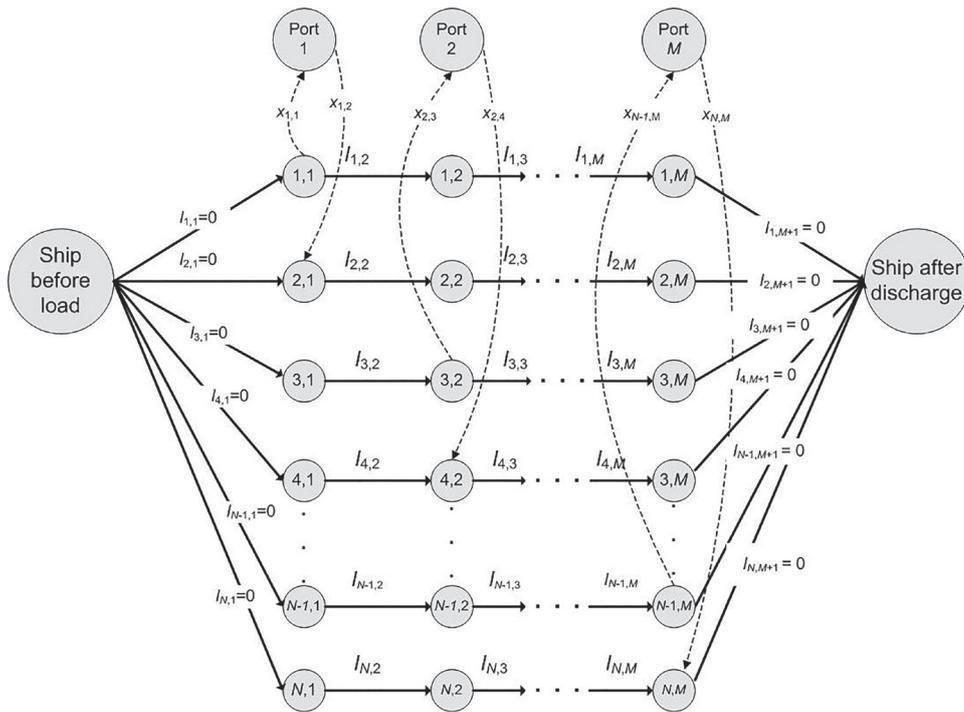


Figure 3 A network flow presentation of the transportation problem  
Slika 3. Transportni problem prikazan pomoću mrežnog prikaza tokova tereta

In the mathematical model form fig. the following notation is used:

$i, j$  and  $k$  = indices for cargo load. The  $N$  facilities are not ranked, just present different types of cargo/container contingents from 1, 2, ...,  $N$ . Odd numbers are for empty and even for loaded containers.

$m$  = indices the port of loading (charging) or discharging. The number of port of calls on the voyage including departure port is  $M$  ( $m = 1, \dots, M$ ).

$x_{i,m}$  = quantity of  $i$ -th load of cargo amounts (e.g. containers contingent) being loaded on board in port  $m$  (TEU).

$Lx_{i,m}$  = limitations for each port and for each cargo load. For convenience, the  $x_{i,m}$  is assumed to be integer.

$l_{i,m}$  = the amount of cargo load  $i$  at arrival in port  $m$  (or, equivalently, at the departure in port  $m-1$ ). Before the first port of loading,  $l_{i,m} = 0$ . After last port  $l_{i,M+1} = 0$  for  $i=1, \dots, N$ . Capacity values cannot be negative.

step  $l_i$  = the lowest step of possible capacity loading and discharging for capacity type  $i$ . In our numerical test-examples it can be set e.g. step  $l_i = 5\%$  of total capacity of the ship.

The complexity of the proposed algorithm is  $O(C_p^2)$ , where  $C_p$  is the number of capacity points; see explanation in [10]. That value is in strong correlation with number of ports  $M$  and number of contingents  $N$  but also with capacity increment step  $l_i$  that can be variable from contingent to contingent.

The network optimization can be divided in two steps. At first step the minimal transportation weights (cost value for transport) between all pairs of capacity points (neighbor ports on the route) are calculated (see equation 3.1). It is clear that many values  $d_{m,m+1}$  that emanate two capacity points of neighbor ports. At second step we are looking for the shortest path in

the network with former calculated weights. The number of all possible  $d_{m,m+1}$  values depends on the total number of capacity points  $C_p$ .

In this research load amount on board do not influence on voyage speed neither to oil consumption but it could be easily incorporated.

### 3. RESULTS OF BASIC HEURISTIC / Rezultati osnovne heurističke metode

In route definition for example from fig. 2 we have starting and ending port 1, but any of four middle ports can also be included in the route. All distances between ports are defined in miles. From figure 2. we can see traffic demands (possible transfer of contingents) given in the percentage of the total ship capacity. In this test-example we do not have contingents between middle ports, only transport related to/from home port, but it is not limitation.

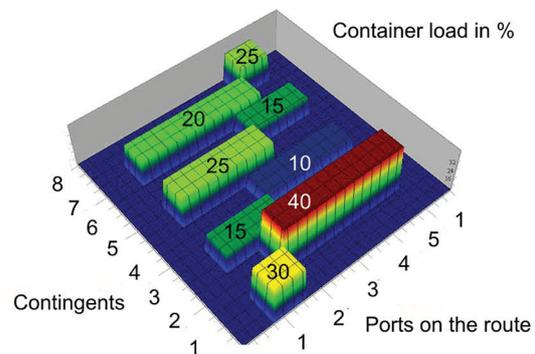


Figure 4. Amount of container contingents on the ship during voyage

Slika 4. Količina kontejnera za pojedini teret u toku plovidbe

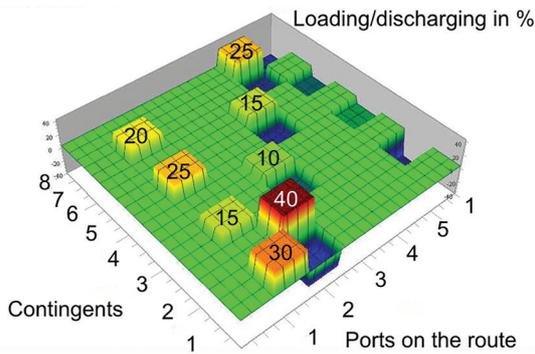


Figure 5 Moments of loading/discharging  
Slika 5. Trenutci krcanja/iskrcaja

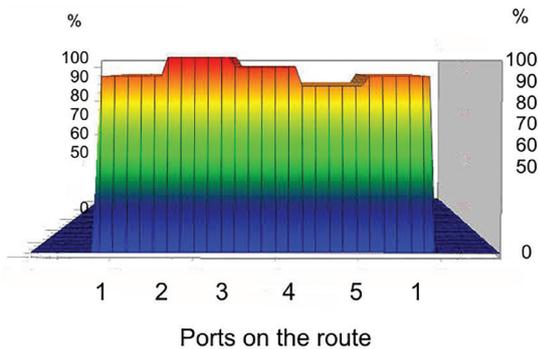


Figure 6 Efficiency of the ship on the route (occupancy)  
Slika 6. Učinkovitost brodskog prostora na putovanju (popunjenost)

If we have demands for empty containers that exceeds the amount that we can take from home port, we can supply it from the neighboring port. This model allows such solution, so cargo manager can include such contingent in account. Only condition is that surplus of empty containers are waiting in one of previous ports. All information about potential load can be gathered through market research or from statistics of port authorities. Basically we have four cost types: freight cost (income), transport cost caused with distances, port taxes and cost of loading/discharging operation (transshipment cost). As the objective function is complex function, consisting of two or more non-linear cost functions (showing effect of economy of scale), it becomes very hard problem. Complexity depends on algorithm type we use [13]. If we have no freight cost, only transport and transshipment costs (including port taxes) influence on optimal solution. In that case we are looking for minimization of equitation (3.1). For simplicity in this research, all costs elements for any contingent and any port are equal, see [17 and 18].

The objective function  $d_{m,m+1}$  can be formulated as follows:

$$\max \left( \sum_{j=1}^N \sum_{m=1}^{M+1} f_{i,m}(I_{i,m}) - c_m(d_m) - g_{i,m}(x_{i,m}) - h_m - \text{idle}(W, I_{i,m}) \right) \quad (3.1)$$

for  $m = 1, 2, \dots, M+1$ ;  $i = 1, 2, \dots, N$ .

As we can see from the equation (3.1) the objective function (total cost) includes some different costs. In fact, we have dual min-max problem. Freight cost is denoted with  $f_{i,m}(I_{i,m})$  and we

have to differentiate freight cost for each container contingent and for each transport distance between ports  $m$  and  $m+1$ . All expenses have negative polarity. It means that profit will be reduced by transportation cost dependent on distances  $c_m(d_m)$  and with transshipment (load/discharge) cost  $g_m(x_{i,m})$  related on loading/discharging amounts. Also, the port taxes can be introduced as  $h_m$  for each port specifically.

The idle capacity cost can be taken in account but only as a penalty cost to force the usage of maximal capacity (prevention of unused/idle capacity). So we have adding cost: *idle* ( $W - I_{i,m}$ ), where  $W$  is total capacity of the ship and  $I_m$  is total cargo load on board for each contingent and for transport distance from port  $m$  to port  $m+1$ .

$$I_m = \sum_{i=1}^{N+1} I_{i,m} \quad (3.2)$$

Costs are often represented by the fix-charge cost or/with constant value and variable part. It should be assumed that all cost functions are concave and non-decreasing (some of them reflecting economies of scale) and they differ from one contingent to another or from one port to another. The objective function (3.1) is necessarily non-linear and exponential. The problem can be seen as looking for maximal value of profit, that is logically in relation to minimal expenses.

If we differentiate freight cost, e.g. higher cost for loaded containers, that cost will be included in objective function with opposite polarity, with intention to increase the profit of transportation. Such dual min-max problem could be solved with maximization of the profit. We can use the same technique of minimization but objective function has to be with negative polarity. Definitely, introduction of different freight cost (higher cost for loaded containers) it will influence on optimal solution and we have quite different results, see fig. 7 – fig. 8.

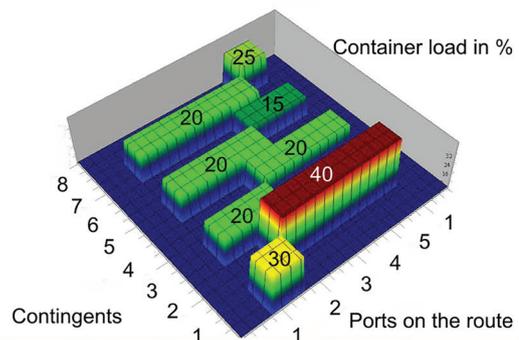


Figure 7 Here we prioritize the transport of loaded containers, so we can see that contingent 4 (3-1) is much higher than before. Now, the fourth contingent is transported in total amount, see difference on fig. 4

Slika 7 Ovdje se potencira prijevoz punih kontejnera, tako je kontingent 4 (3-1) mnogo veći nego u prethodnom primjeru. Sada je četvrti kontingent prevezen u punom iznosu, u usporedbi sa slikom 4

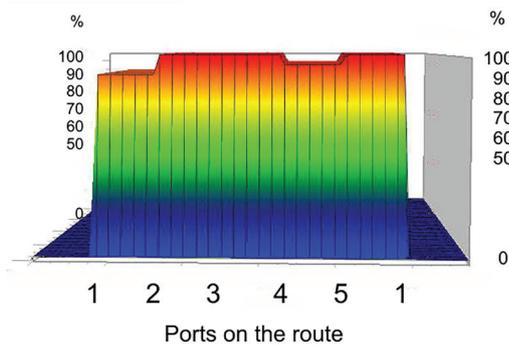


Figure 8 Ship's occupancy is closer to 100 % than in previous situation from figure 6

*Slika 8. Popunjenost broda je bliža 100 % nego na slici 6*

According to all transport costs elements (freight cost, oil consumption, transshipment cost, port taxes etc.) we can design the route which will be the most profitable. In figures 7. and 8. the resulting (the best) route is presented. Fig. 7. shows the load amounts of every contingent on board during the voyage. We can see that contingent 4 (3-1) is much higher than before. Fig. 8. shows that ship's occupancy rises, too. For the basic algorithm option we used the same capacity increment  $step I_i$  for all contingents and it is 5%. Such capacity resolution can be satisfactorily if we have small capacity of the ship, e.g., for 40 TEU (40 x 20 foot containers), so the capacity increment is 2 TEU, that is equal to two containers (2 x 20 ft). But if we have 100 TEU ship, the capacity increment is 5 TEU, so the calculation can be far away from optimal result. In that case we have to decrease  $step I_i$  on value of 2% or 1% (not less the value that means one container). Normally, it causes huge number of capacity points and much higher complexity, so calculation duration drastically rises. So it could be the problem for limited calculation power.

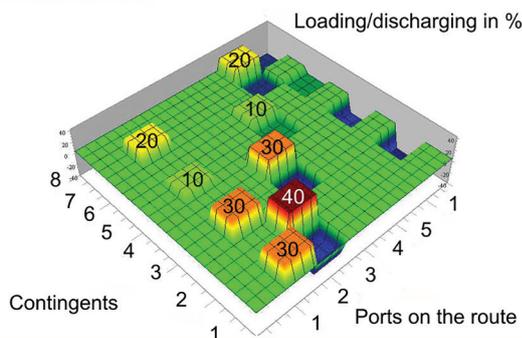


Figure 9 The main part of cargo are contingents that stay on ship longer (better ship's occupancy), but contingents closer to destination (home) port are lower in amount. Problem of voyage direction can appear, but it can be solved with different port numeration

*Slika 9. Glavnina tereta su kontingenti koji putuju duže (bolja popunjenost broda), ali su zato kontingenti kojima je polazište bliže krajnjem odredištu manje zastupljeni. Može se postaviti i problem smjera obilaska luka, što je samo stvar brojčanog označavanja luka*

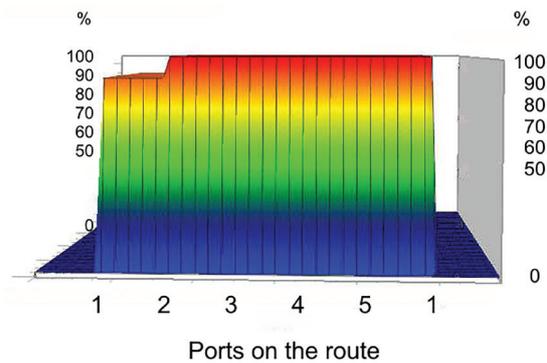


Figure 10 Ship's occupancy for third example is maximal  
*Slika 10. Popunjenost broda je ovdje maksimalna*

Also, the complexity is firmly dependent of number of ports on the route  $M$  and with number of contingents  $N$ . If we have very huge problem we can solve it in steps of calculation, consisting of successive iterations that can decrease the complexity to acceptable level.

In the third example we can force the maximal loading (occupancy) of the ship. In that sense we can introduce penalty cost for idle capacity, see (3.1). From fig. 9 and fig.10. it is clear that result is quite different, the idle capacity of the ship is lower. No better ship's occupancy exists for this example, but it depends on all cost elements and their relations. In such case, where we introduce penalty cost for empty space, it can cause non-optimal transport (not maximal profit).

Through many test examples it is clear that such approach functions good and calculation complexity of the optimization process is under control. Such planning tool offers a lot of possibilities for modeling to final result, that is firmly connected with knowledge of cargo officers. So they have to be well trained to use such optimization tool.

#### 4. CONCLUSIONS / Zaključci

The proposed algorithm shows ability to solve very demanding transportation problem with many loading/unloading ports and with various container contingents. In distribution of loaded containers from many small ports to home port it could be very useful to optimize efficiency of cargo space and reduction of transport costs, resulting in higher profit. In the same time the transport of empty containers has to be optimized, too. With such planning tool the cargo officers (on board or in small port) is supported with capable tool for decision making, to be able to satisfy traffic demands and easily adapt to its changes. Sometimes the transportation has to be the most profitable option, but not always.

Also, the limited calculation power in shipping surrounding doesn't support high algorithm complexity. So this approach can solve very huge problems in steps, consisting of successive iterations that can decrease the complexity to acceptable level. In the same time it ensures the cargo officers on board a very fine optimization tool, enabling many input values and leading the optimization process in wanted direction.

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