

# Travelling Salesman Problem Applied to Black Sea Ports used by Czech Ocean Shipping Companies

## Primjena problema trgovačkog putnika na crnomorske luke na primjeru čeških brodarskih tvrtki

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DOI 10.17818/NM/2018/3.2

UDK 656.61:330.1 (437.3)

Preliminary communication / Prethodno priopćenje  
Paper accepted / Rukopis primljen: 15. 11. 2017.

### Summary

Graph theory offers useful tools for solving problems in transportation. This article concerns the Travelling Salesman Problem. This classic transport problem is addressed in terms of Czech shipping companies and the ports on the Black Sea. Using mathematical software, a Hamiltonian cycle with the smallest sum of the weights of the edges along these ports is found and discussed. Algorithms based on graph theory are used to find the economically most advantageous path. The start and end of the route are located in Prague because Czech companies currently operating in maritime transport have headquarters located there.

### KEY WORDS

czechoslovak ocean shipping  
travelling salesman problem  
graph theory  
Hamiltonian cycles

### Sažetak

Teorija grafova nudi korisne alate za rješavanje problema u prijevozu. Članak se bavi problemom trgovačkog putnika. Ovaj klasični transportni problem stavljen je u okvir čeških brodarskih tvrtki i crnomorskih luka. Primjenom matematičkog softvera pronađen je i razmotren Hamiltonov ciklus s najmanjim zbrojem težina na bridovima ovih luka. Koriste se algoritmi koji se temelje na teoriji grafova kako bi se pronašla najekonomičnija putanja. Početak i kraj rute jesu u Pragu jer se ondje nalaze središta čeških brodarskih tvrtki.

### KLJUČNE RIJEČI

čehoslovačke brodarske tvrtke  
problem trgovačkog putnika  
teorija grafova  
Hamiltonovi ciklusi

## 1. INTRODUCTION / Uvod

The process of travelling and transporting people, animals and material has been the concern of humanity since the beginning of civilisation. It is therefore not surprising that there have been attempts to optimize transport from an economic point of view. One available method is the application of the Travelling Salesman Problem (TSP). For definition of the transportation problem in maritime we have to develop appropriate mathematical model represented with weighted graph where vertices represent sea ports and the edges represent the routes between the ports through which ships transporting goods or people. Here, the solution is applied to maritime topic – the optimisation of travelling along individual ports recently used by Czech shipping companies on the Black Sea. The goal of optimisation is to reduce travel costs. A shorter route obviously saves fuel and therefore financial resources.

Chapter 2 describes Czech naval history, especially the origin, development of the Czechoslovak Ocean Shipping company (COS) and gives the list of its home ports, located mainly in former member states of the Council for Mutual Economic Assistance (CMEA), countries allied with the former Czechoslovak Socialist Republic. Apart from providing access

to international sea lanes, these ports also met the condition of being easily accessible from Czechoslovakia via land routes. These ports are used by Czech transport companies even in the present.

Chapter 3 is devoted to the formulation and description of the Travelling Salesman Problem by tools of graph theory. There is included a brief historical summary of TSP and theoretical basement of Hamiltonian paths. A considerable progress is apparent in solving this problem owing to rapid development of computing technology.

Chapter 4 addresses a specific maritime traffic problem - an optimisation of the round-trip leading from Prague through the Black Sea ports. In addition to optimising the desired route, it contains a discussion of some interesting phenomena encountered during the calculations. The main result is the identification of the economically most advantageous path.

## 2. HISTORY OF CZECH OCEAN SHIPPING / Povijest čeških brodarskih tvrtki

The watercourses on the territory of today's Czech Republic have served as important transport arteries since ancient

times. A regular freight river transport is attested since the 11<sup>th</sup> century. Important trade routes passed through river courses to seaports. In the Middle Ages, there appear first mentions of maritime navigation related to the Kingdom of Bohemia. The Kingdom of Bohemia was, owing to border changes, a maritime state through part of its history even before becoming part of the Habsburg Monarchy.

The Habsburg Monarchy was a maritime state with its own ports and navy. At the time when the Kingdom of Bohemia was part of the Austrian (later Austro-Hungarian) monarchy, many Czech and Slovak citizens served on warships. After the dissolution of the monarchy in 1918, Czechoslovakia formed as a landlocked country without a naval fleet. The origins of Czechoslovak maritime shipping are tied to the historical origin of the Republic of Czechoslovakia in 1918.

The Treaty of Versailles (1919) and the subsequent Barcelona Declaration allowed the use of national flags on seagoing ships even for countries without their own coastlines. Prague became the port of registry for Czechoslovakia and ports in Stettin and Hamburg (Moldauhafen) were leased from Germany for a 99 year-period. The first ship sailing under the flag of Czechoslovakia was the schooner Kehr wieder, in 1920. Seagoing ships were owned and operated by various private companies, e. g. Baťa Shoe Company [6]. In the interwar period, 80 Czechoslovak citizens studied at the naval academy in the then Yugoslavia. Thanks to this, post-war Czechoslovakia gained qualified and experienced naval officers and sea captains. Czech and Slovak sailors, from naval officers to ordinary crew members, have always had excellent technical education.

During the World War II, Czechoslovakia was divided for the first time. Bohemia and Moravia (the present Czech Republic) fell under the direct rule of Nazi Germany, while Slovakia became a separate state under German influence. During this period, there was no development of Czechoslovak maritime shipping.

After the end of World War II, the Czechoslovak Republic was re-established and, in 1948, joined the socialist bloc countries. Foreign trade was one of the socialist countries' priorities, which enabled the establishment of state-managed Czechoslovak maritime shipping. The year 1958 saw the founding of the Čechofracht company, whose activities included international transport. A year later, a state-owned joint-stock company called Czechoslovak Ocean Shipping (COS) was established. Details of COS history are discussed, for example, in [3], [9] and [10].

The COS was a profit making organisation throughout its existence. At one time or another it operated 44 cargo ships of different classes and sizes, including a tanker (almost 950 thousand DWT altogether). Most of these ships were owned by the COS, a smaller part was owned by China. The COS fleet was consistently one of the largest among those operated by landlocked countries. In 1990, Czechoslovakia had the 74th largest merchant fleet in the world by DWT, which even amounted to being first among landlocked countries [3]. During its existence, the COS was using the following ports:

#### *Baltic Sea*

Gdansk, Gdynia, Swinoujscie, Szczecin, Wismar, Rostock, Lübeck

#### *North Sea*

Hamburg, Bremen, Bremerhaven, Emden, Amsterdam, Rotterdam, Antwerp

#### *Adriatic Sea*

Koper, Rijeka, Bakar, Ploče, Omišalj, Trieste, Venice

#### *Black Sea*

Braila, Galati, Reni, Izmail, Constanza, Varna, Burgas

In 1992, the company was privatised. Its name was changed to Czech Ocean Shipping in 1994, because on 1. 1. 1993 Czechoslovakia split into two independent states – the Czech Republic and the Slovak Republic. Although its shipping business had been consistently profitable, ships in its inventory were gradually sold off and thus the company was in a profound loss by 1998. It stopped operating seagoing ships. Some of its subsidiaries, e. g. C.O.S. - Crew Management were sold off [3]. The C.O.S. - Crew Management currently provides work for Czech and Slovak sailors, for example on Italian and German ships (often in the position of naval officers).

However, the termination of COS in 1998 did not mean the retreat of the Czech Republic from the position in world shipping it had held so far. The role of the defunct company was partially taken over and partially complemented by newly created firms. There are currently several transport companies operating in Czech Republic that engage in ocean shipping. These are, for example, DSV AIR & SEA s.r.o., Cargo IHL - International Shipping Company, Multitrans CZ s.r.o., Logex Logistics, s.r.o. These companies are represented in most of the major world ports, including former home ports of COS.

### **3. GRAPH THEORY AND TRAVELLING SALESMAN PROBLEM / *Teorija grafova i Problem trgovačkog putnika***

#### **3.1. Travelling Salesman Problem (TSP) / *Problem trgovačkog putnika (TSP)***

The Travelling Salesman Problem (TSP) is formulated as a task with a given set of cities and the routes between them. The task is to find the shortest (most economical) route passing through all the cities and returning to the starting point (home port). TSP is usually formulated using concepts based on graph theory. Graph vertices represent ports and cities, graph edges represent individual routes between ones. The solution of the problem lies in finding a closed path (a circle) in a weighted (usually non-oriented) graph.

It is clear from the very nature of the task that computational software can be used satisfactorily for its examination. Today, we have many applications in many programmable tools. The program named "Concorde" is considered to be the most efficient tool for solving the TSP. Its source code is 130 thousand lines long and is publicly available ([www.tsp.gatech.edu](http://www.tsp.gatech.edu)). This computational tool is routinely used for solving tasks with vertices numbering into tens of thousands. Also, this approach has no limitation if A – B distances are not the same in both directions.

However, the beginnings of algorithmic optimisation of travel between cities have a much longer history [1]. Among those who demonstrably concerned themselves with this problem was e. g. Abraham Lincoln on his round trip along fourteen law courts in the state of Illinois, in 1850. Today, we know that Lincoln's route was not the optimal one and could have been shorter, which naturally does not make the story less interesting. Another example from the same era would be

Mark Twain's book *Innocents Abroad*, in which he encounters the problem of finding the shortest steamboat route along the ports of the Mediterranean. This book was even reportedly his best-selling work during his lifetime.

The research of the TSP extends throughout the 20th century, during which researchers attempted to minimise the route for an increasing number of cities, up to the above-mentioned Concorde program. The advances in solving the TSP can be seen in the increasing number of vertices that individual authors were able to compute in their time. It is possible to observe a direct dependence between the advances in computational technology and the complexity of tasks being solved.

In the 1950s, the Travelling Salesman Problem was solved for 49 vertices. At the beginning of the 1980s the number stood at 2392 vertices. In 2004, the shortest circular route was found for 24,978 sites in Sweden (<http://www.math.uwaterloo.ca/tsp/sweden>). For finding the solution, 96 IntelXeon2.8 CPUs were used, and the calculations took more than a year. Current state-of-the-art technology is able to solve even more complex tasks than the above-mentioned Swedish one. The Travelling Salesman Problem therefore remains a valid and open issue. A more detailed analysis of the Travelling Salesman Problem can be found in summarizing works [1], [4] and [13].

Here we mostly discuss about single-commodity transportation problem, but in cargo transport we often need multi-commodity approach, that is more complex. Also, linearity of transport costs is not present in practice so often, so non-linear problems could be more demanding [12].

### 3.2. Graph Theory and Hamiltonian Cycles / Teorija grafova i Hamiltonovi ciklusi

All graphs represented in this article are finite, connected, weighted and closed with oriented edges. The system of traffic routes we can transform into the graph where vertices represent sea ports, edges represent transport routes and weights of edges represent the energy consumed to drive the transport mean (ship, car, airplane, train or something else) between two ports. To model this situation we create a connected graph  $G=(V,E)$ , where  $V$  is the set of  $n$  vertices and  $E$  is the set of edges as usual. The edges are weighted.

For application of TSP algorithms first we have to create Weighted Adjacency Matrix. This matrix is similar to the Adjacency Matrix where in positions of elements of the matrix are either 1 or 0 if there is an edge between vertices  $v_i$  and  $v_j$  or not. In the Weighted Adjacency Matrix the positive number  $w_{ij}$  on the position of the element  $v_i$  and  $v_j$  indicates the weight of the edge connecting vertices  $v_i$  and  $v_j$  if the edge between vertices  $v_i$  and  $v_j$  exists. A value of 0 indicates that there is no

edge between vertices  $v_i$  and  $v_j$  (see Figure 1),  $i,j \in \{1, \dots, n\}$ .

Weighted Adjacency Matrix is a square matrix which is usually assumed symmetric with respect to the main diagonal. In our case (see Table 1) we can see that the matrix is not symmetric due to different distances in opposite directions. The fact that matrix is not symmetric has no influence to used algorithm.

A Hamiltonian path is a path that visits each vertex of the graph exactly once. A Hamiltonian cycle is a closed path that visits each vertex exactly once (except for the vertex that is both the start and end, which is visited twice). A graph which contains a Hamiltonian cycle is called a Hamiltonian graph. In the language of graph theory the TSP means to find the Hamiltonian cycle with the smallest sum of the weights of the edges. For solving the problem the linear programming methods are used, among others Cutting-plane method. The algorithm starts with an admissible solution for set of edges  $S$ . Then the optimal solution is found by the simplex method. If this optimal solution is not contained in the set  $S$  then the optimal solution can be separated from  $S$  by a hyperplane and the equation of the hyperplane is added to the restrictive conditions of solved problem. Further, iterate this process until an optimal solution is found (for more details see [1]).

In Chapter 4, Hamilton cycles are evaluated according to some additional criteria. The shortest (or the longest) paths in different directions are compared here. Thus, it is not enough to find the shortest path that does not take these criteria into consideration. For this reason, the algorithm used here is different from cutting planes and it goes through all possible cycles regardless of the Cutting-plane method. Then the comparison of paths in terms of their length and the direction is following. The Hamiltonian cycles are found by computational method with mathematical software.

### 4. APPLICATION OF THE TSP TO THE ROUND-TRIP ALONG THE PORTS / Primjena TSP-a na kružna putovanja

Czech companies currently operating in maritime transport have headquarters located in Prague or its immediate vicinity. Sales representatives of these companies visit ports in various locations. We assume a model situation, in which we select ports, that are located in the Black Sea area (Braila, Galati, Reni, Izmail, Constanza, Varna, Burgas). Those ports are traditionally used by Czech shipping companies. From these, 4 are river ports on the Danube and the remaining 3 are seaports. A representative of company intends to visit all these home ports, his starting and terminal point being Prague. In line with the TSP principles, we search for the shortest route matching the assignment.

	$v_1$	...	$v_i$	...	$v_n$
$v_1$	0	...	$w_{i1}$	...	$w_{n1}$
...	...	0	...	...	...
$v_i$	$w_{i1}$	...	0	...	$w_{in}$
...	...	...	...	0	...
$v_n$	$w_{n1}$	...	$w_{ni}$	...	0

Source: authors

Figure 1 General Weighted Adjacency Matrix  
Slika 1. Opća ponderirana matrica susjedstva

Table 1 The shortest road distances in km

Tablica 1. Najkraće cestovne udaljenosti izražene u km

	Prague	Varna	Burgas	Constanza	Izmail	Reni	Galati	Braila
Prague	0	1279	1304	0	0	0	0	0
Varna	1279	0	0	160	438	372	349	332
Burgas	1304	0	0	290	502	436	414	393
Constanza	0	158	288	0	283	217	194	178
Izmail	0	435	503	283	0	66	89	111
Reni	0	369	437	217	66	0	23,3	44,4
Galati	0	346	414	194	89	23,3	0	21,5
Braila	0	330	392	177	110	44,6	21,7	0

Source: authors, distances from <https://mapy.cz>

For simplicity, we assume that the trip from Prague to the Black Sea area takes place by air, the representative then travels through the individual ports and returns, again by air, to Prague. Given that among the cities in question only Varna and Burgas have international airports with direct flights from Prague, these two locations necessarily become the second and the penultimate vertices in our graph. Therefore the optimal route can be either

Prague – Burgas ..... Varna – Prague

or

Prague – Varna ..... Burgas – Prague.

For calculating and finding the shortest route we are going to use the road distances between the ports listed in Table 1 and the air distances from Prague to the Black Sea.

Simultaneously, Table 1 represents the Weighted Adjacency Matrix according to the general definition in the chapter 3.2. (see Figure 1).

In most theoretical tasks of this type, it is assumed that the distances between two vertices A and B are the same in both directions  $A - B$ ,  $B - A$ . However, in case of a real road traffic, these distances may be different (see Table 1). The difference may be caused, for example, by one-way roads, highway connections, city bypasses, etc. If we choose to take this into account, the task becomes more complicated and, in our opinion, also more interesting. However, a greater caution is also needed while approaching the problem in this manner. Intuitive thoughts might fail and it is necessary to examine the route in both directions.

Using the “Maple” mathematical software, the following information has been obtained.

#### 1. Prague – Burgas

In this direction, the shortest route found has the length of 3527,3 km. The cities were passed in the following order: Prague – Burgas – Braila – Izmail – Reni – Galati – Constanza – Varna – Prague (see Figure 3). Due to asymmetries in Table 1, the length of this route in the opposite direction is 3529,3 km.

#### 2. Prague – Varna

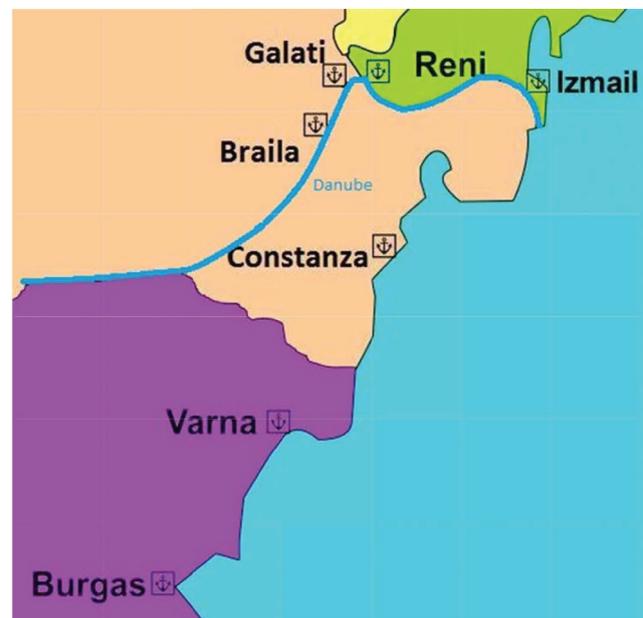
In this direction, the shortest route has the length of 3528,4 km. The cities were passed in the following order: Prague – Varna – Constanza – Galati – Izmail – Reni – Braila – Burgas – Prague. The length of this route in the opposite direction is 3527,6 km.

The routes in which Varna is the second vertex are longer than the routes starting with the Prague – Burgas step. The shortest route overall is therefore 3527,3 km long. Just for interest, the longest possible route would measure 4001,7 km and there are even two different routes of this length:

Prague – Varna – Reni – Constanza – Izmail – Braila – Galati – Burgas – Prague,

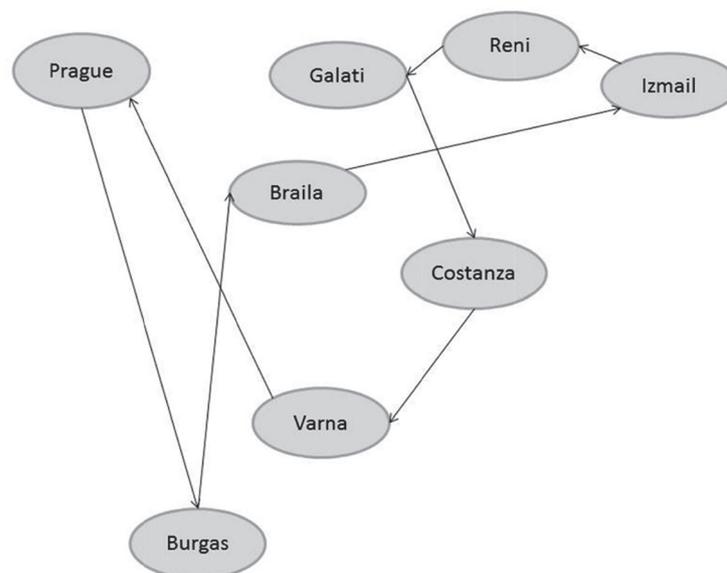
Prague – Varna – Izmail – Braila – Galati – Constanza – Reni – Burgas – Prague.

Looking at the geographical map (Figure 2), we might say that the shortest route as it was actually calculated and found would be probably different from the one that might be chosen intuitively. The traveller would most likely choose to visit the Danube ports successively passing either in the direction upstream or downstream along the Danube (Braila – Galati – Reni – Izmail or Izmail – Reni – Galati – Braila), which would not be the optimal solution.



Source: <http://tripmeter.me>

Figure 2 Distribution of marine ports  
Slika 2. Raspored morskih luka



Source: authors

Figure 3 The shortest closed path

Slika 3. Najkraća zatvorena putanja

The difference between the shortest and the longest route is 474,4 km. It is clear that the differences between existing routes can be significant, therefore arranging the cities on the route in the right order can bring considerable economies.

## 5. CONCLUSION / *Zaključak*

Today, alternative transport options that might help relieve congested road traffic [7] are becoming increasingly important. Thus, international shipping is undoubtedly significant from a global pan-European point of view.

Solving the task regarding the Black Sea ports has provided some new insights and interesting points. The shortest (i. e. most economically viable) route for inspection of the ports has been found. The actual road distances between the individual cities are not the same in both directions (see Table 1). This asymmetry is one of the main factors that make the optimal solution unlikely to be found at the first glance.

The shortest route (Prague – Burgas – Braila – Izmail – Reni – Galati – Constanza – Varna – Prague) is 3527.3 km long (see Figure 3). We might note that passing the Danube ports intuitively in the direction downstream would make the route (Burgas – Braila – Galati – Reni – Izmail – Constanza – Varna) longer by 0,7 km. In this case, the difference is negligible and interesting rather from a theoretical point of view. However, in the case of travelling over longer distances, the difference between “intuitive” and optimal solutions may be significant. Therefore, solving tasks involving the TSP while accounting for different distances in both directions is also justified in practical terms.

There are other possible approaches to economic and logistic optimisation than the TSP. One of these may be to examine the parameters that are needed to determine the most suitable location of transport or logistics centres [8] or relevant for traffic situation modelling [2], including passenger transport [5]. Using graph theory, this approach studies not the graph edges, but the location of the vertices. Another possible view on economic optimisation of freight transport is discussed in, for example [11] and [12].

## ACKNOWLEDGMENT / *Zahvale*

This article was supported by project grant No. IGS06C2, Faculty of Economics, University of South Bohemia, Czech Republic.

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