Particle's Trajectory – Implementation in Maritime Traffic

Trajektorija čestice – primjena u pomorstvu

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Summary

The paper addresses the application of the mathematical definition of a particle's trajectory in maritime traffic. After the introductory definition of the concept of a particle's trajectory, it goes on to provide a definition of the concept necessary for the understanding of the issues in question: the concept of the manoeuverability. Next, an example is given which defines the concept of a wave particle's trajectory and the method of ship manoeuvring along a trajectory. Finally, there is a definition of the trajectory error and the method how to calculate it. There is also following the description of block scheme which depicts the way of steering a ship along a given trajectory with corresponding transfer functions. At the end, there is a conclusion.

KEY WORDS
the particle
the particle's trajectory
the wave
ship's motion
the trajectory error

Sažetak

Rad opisuje primjenu matematičke definicije trajektorije čestice u pomorstvu. Nakon uvodnog dijela u kojem se definira koncept trajektorije čestice, kao i njena točna definicija uz prikaz nasumičnog hoda čestice kroz medij, nastavlja se s definiranjem pojma koji je nužan za razumijevanje daljnje materije, pojma pomorstvenosti. Dalje je detaljno opisan pojam trajektorije čestice vala te metoda vožnje broda po zadanoj trajektoriji. Matematički model vožnje broda po zadanoj trajektoriji definira i grešku trajektorije, te način na koji se ona može izračunati. Na kraju se nalazi opis blok shema koje opisuju način vožnje broda po zadanoj trajektoriji uz odgovarajuće prijenosne funkcije. Nakon svega slijedi zaključak.

KLJUČNE RIJEČI
čestica
trajektorija čestice
valovi
gibanje broda
greška trajektorije

1. INTRODUCTION / *Uvod*

The life history of a particle is built up from an acquirement of its trajectory through the particular system of interest. The concept of trajectory has several meanings. It can denote a curve which describes the movement of a planet or comet in space; it can denote a path or progression or the line of a material object movement, a physical body. Therefore, a particle's trajectory describes the path that a certain particle has travelled in the course of its life path.

In this article, different implementation of the term of trajectory is described. In fact, the term trajectory has a different meaning in different applications.

2. THE TRAJECTORY / *Trajektorija*

The history of particles is developed from the moment of comprehending their trajectories to the elaboration of a system of particles that in itself is of great significance. [12]

One of the descriptions of the particle´s trajectory is depicted down.

The concept of the path of a particle is describing its travelling through the medium. [1] [7] Since the typical particle scatters very frequently, the path has „zig-zag“ form indicated in Fig. 1.

The particle has a collision with an atom of the medium. That mode of a collision could result in the absorption of the particle and its ending/termination or a particle continues its travelling through the medium with a new direction and a change of energy. Since the change of energy and direction is a statistical process, there is not a unique energy or direction after the scattering. In fact, there is a probability distribution for each of these variables. After the first scattering, a particle has a new direction and energy, it experiences another collision after which it has a new direction and energy, etc. That process is well described in Fig. 1. In order to track the particle during its journey, it is required to know the following quantities: its spatial coordinates (x,y,z), the spherical coordinates of its direction (θ, Φ) and its energy E.

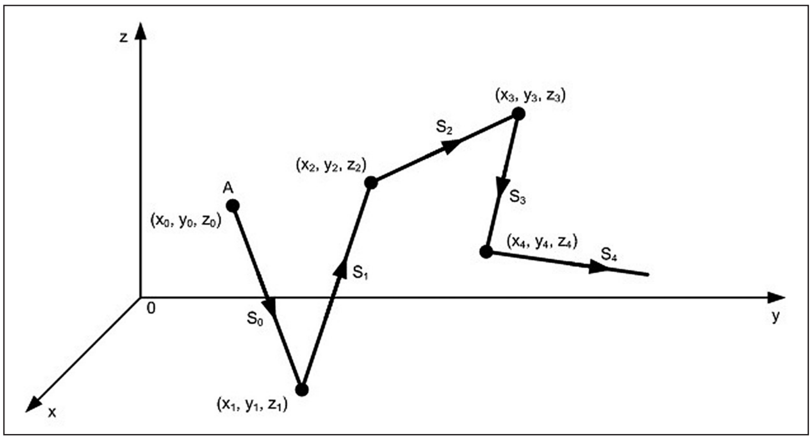

Figure 1 Characteristic "random walk" of a particle through the medium
Slika 1. Karakteristični "nasumični hod" čestice kroz medij

The orthogonal coordinate system (*x′,y′,z′*) is parallel to the basic reference system (*x,y,z*) shown in Fig. 1.

The quantities above are sufficient to define the state, α, of a particle, where:

$$\alpha \equiv \alpha(x, y, z; E; \theta, \Phi) \quad (1)$$

A particle's trajectory from collision to collision is depicted like a series of the different states, α_0, α_1, ... α_n. Hence, α_i described i^{th} state of a particle and it is defined by the equation:

$$\alpha_i \equiv \alpha_i(x_i, y_i, z_i; E_i; \theta_i, \Phi_i) \quad (2)$$

What means in the i^{th} state is that a particle has spatial coordinates of the ith collision point, the energy and direction of a particle after the i^{th} collision. Each successive state is a function only of the previous state. The exception is the initial state. Conditions which define α_0 are chosen by random sampling from the relevant probability distributions [1].

2.1. THE PARTICLE'S TRAJECTORY IN MARITIME TRAFFIC / *Trajektorija čestice u pomorskom prometu*

The manoeuverability of the vessel is a term which analyses response on sea's waves to estimate capability of the vessel and her safety on rough sea. [2] [17]

Namely, ocean going vessels are constructed in such a manner so that they can operate in a sea state which is very often unpleasant, frequently unbearable and dangerous. The ship roll dynamics has been described using roll equations which balance the external forces and moments that affect the vessel with the internal forces and moments due to inertia [2].

The Swiss mathematician Leonhard Euler, French astronomer Pierre Bouguer and Swiss scientist Daniel Bernoulli tried to formulate and solve the ship roll equations in the 17th century, and their attempts are considered the beginning of the sea keeping theory (the theory of manoeuverability). [15] [16] [10] [11] [12]

Pauling and Wood developed a simulation program of ship rolling with six degrees of freedom.

Since shifts and loads in extreme sea conditions are crucial for the safety of the crew and the ship, this problem has received a lot of attention and the development of the theory of manoeuverability marks its greatest advances precisely in this field. [13] [14]

As one can notice, the concept of sea keeping theory has,

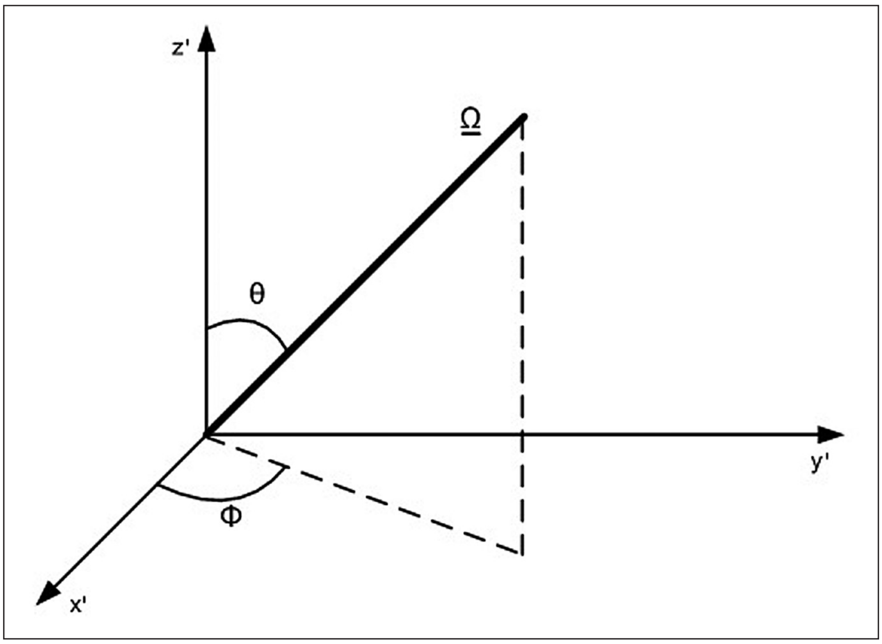

Figure 2 Spherical coordinates of the particle direction (*θ, Φ*)
Slika 2. Smjer čestice u sfernim koordinatama

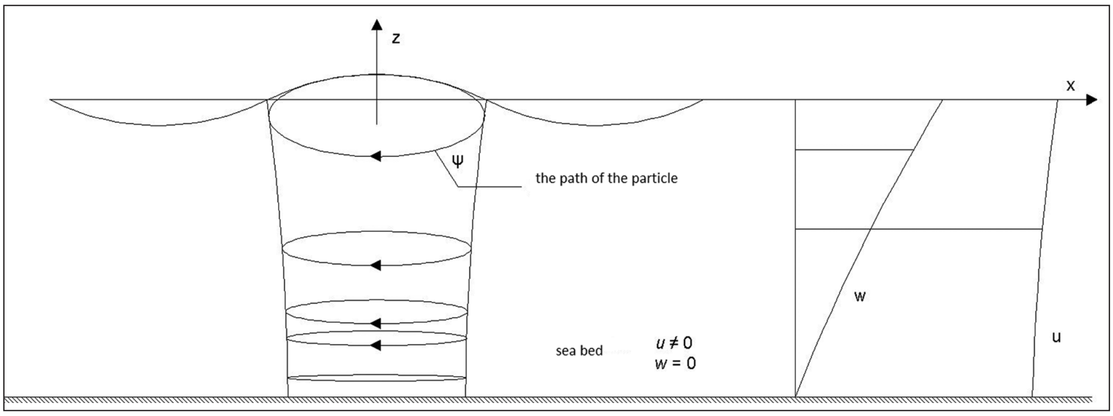

Figure 3 A wave particle's trajectories in the fluid whose depth (for example, sea bed) is limited
Slika 3. Trajektorije vala čestice u tekućini ograničene dubine

especially in the last three decades, become of interest to many engineers, mathematicians and physicists. Their research, along with the currently ongoing research, has turned the sea keeping theory into a useful means of predicting the ship's motions on the waves. Thanks to the common efforts of oceanographers, mathematicians, physicists and engineers, today a ship's designer can, with the help of theoretical and numerical procedures, analyse the ship's behaviour in different wave conditions. Nevertheless, the sea keeping science has many under-researched areas, which pose a challenge to the present and future scientists.

The term trajectory has two different meanings and so, two different applications in the theory of maneuverability. One is describing the type of wave and the other one is depicting the driving. [2]

2.1.1 A WAVE PARTICLE'S TRAJECTORIES / *Trajektorije vala čestice*

If we assumed that the start position of a fluid particle in a wave is the point (x_1,z_1), then (x-x_1) and (z-z_1) are the particle's shifts in relation to that position. Assuming the low wave steepness, these values are low enough for the differences in velocity, which are the consequence of a change in the position, to be neglected, since they are very low and their square is small indeed (negligible) [2].

A new concept is defined - elevation. Elevation or the rising of a free surface is marked with a label ζ. It is described as a harmonic wave of the amplitude ζ_a which is defined as the vertical distance from extreme movements of free surface (a crest or a trough) to the level of calm water. Further on, we define the velocity components that are obtained from the velocity potential. After integrating the equations of horizontal and vertical velocity components, the variable of time is eliminated.

The depth of the sea (water) is marked as d.

The frequency of the wave is marked as ω.

Wave's number is marked as k.

A particle's trajectory has the equation (3).

$$\frac{(x-x_1)^2}{\left(-\zeta_a \frac{\cosh k(d+z_1)}{\sinh kd}\right)^2}+\frac{(z-z_1)^2}{\left(\zeta_a \frac{\sinh k(d+z_1)}{\sinh kd}\right)^2}=(\sin(kx_1-\omega t))^2+(\cos(kx_1-\omega t))^2 \tag{3}$$

The equation (3) can be written in more general case. Then, there is a equation (4) for elliptic paths of the particles:

$$\frac{(x-x_1)^2}{\left(-\zeta_a \frac{\cosh k(d+z_1)}{\sinh kd}\right)^2}+\frac{(z-z_1)^2}{\left(\zeta_a \frac{\sinh k(d+z_1)}{\sinh kd}\right)^2}=1 \tag{4}$$

From the picture 3. it is visible that particle motion dies out with depth.

If the surface of the fluid (sea or water) is calm and free, that implies z_1 = 0. It means that the half of the vertical axis of the ellipse is equal as the amplitude of the wave ζ_a. On the bottom of the fluid (for example sea bed) particles just go in the horizontal direction, meaning $z_1 = -d$. The trajectories of the particles are horizontal lines. [2]

If the fluid is very deep, it is valid:

$$(x-x_1)^2+(z-z_1)^2=\left(\zeta_a e^{kz_1}\right)^2 \tag{5}$$

Hence, the particles of the fluid play along the circular paths with radius of $\zeta_a e^{kz_1}$. Their paths are showed in Fig. 4.

Wave length is marked asλ.

When the depth of the fluid is equal to half of the wave length, the motion of the particle can be assumed to be negligible. For comparison, there are examples of the radius at three different depths:

$$r=\zeta_a e^{kz}=\zeta_a e^{k0}=1{,}0\zeta_a \quad \text{for} \quad z=0 \tag{6}$$

$$r=\zeta_a e^{-k\frac{\lambda}{2}}=\zeta_a e^{-\Pi}=0{,}043\zeta_a \quad \text{for} \quad z=-\frac{\lambda}{2} \tag{7}$$

$$r=\zeta_a e^{-k\lambda}=\zeta_a e^{-2\Pi}=0{,}002\zeta_a \quad \text{for} \quad z=-\lambda \tag{8}$$

The particle velocity vector is a constant absolute value, and it is tangent to the trajectory of the particle.

Here is described the model of particle motion based only on deterministic (not taking into account the random processes) method. It is reflected just the random nature of the waves on sea surface.

2.1.2. STEERING A SHIP ALONG A GIVEN TRAJECTORY / *Upravljanje brodom duž dane trajektorije*

The development of new technologies connected to the exploitation of the underwater world, the use of sea and underwater world for traffic purposes, as well as the progress of mathematics, have led to a new way of steering a ship - sailing along a given trajectory.

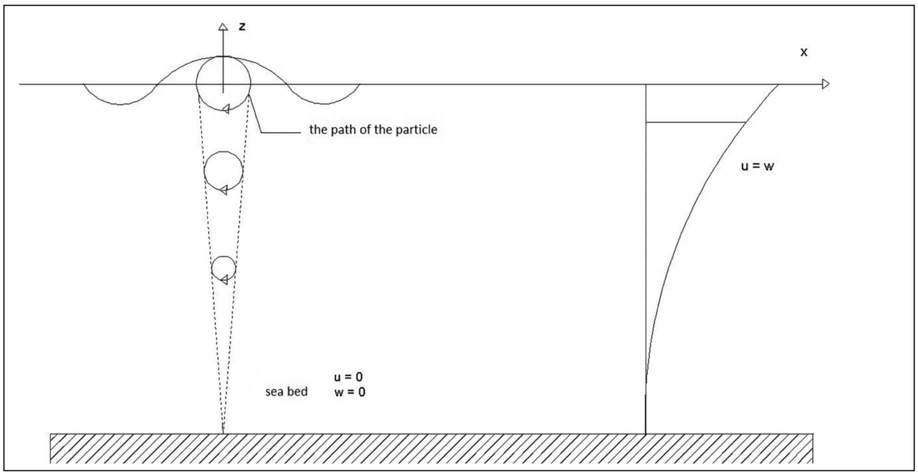

Figure 4 A wave particle's trajectories in the fluid whose depth (for example, sea bed) is unlimited

Slika 4. Trajektorije vala čestice u tekućini neograničene dubine (npr. morsko dno)

Regarding the theory of manoeuverability, the vessel is treated as a solid particle with six degrees of freedom in motion. They are shown in Fig. 5.

0,x,y,z – unfixed coordinate system of a ship
V – the vector of speed of a ship
u – the speed of progress along x-axis
v – the speed of progress along y-axis

Where are:

φ - the angle of the shift around x-axis (rolling)

θ - the angle of the shift around y-axis (pitching)

ψ - the angle of the shift around y-axis (yaw)

β - the angle of yawing of the ship

Ψ – the angle of ship`s course (the yawing)

δ - the deflection of ship`s rudder

V_X - the speed of the ship along x-axis

V_Y - the speed of the ship along y-axis [2]

While a ship [2] is moving on a given course or trajectory, the assignment of control of a ship is to control the moving her centre of mass in horizontal level, i.e. the analysis of moving the solid element with three degrees of freedom:

Moving the ship with speeds V_x i V_y along x-axis and y-axis with starting point in the centre of gravity of the ship

Moving the ship around vertical axis using angle`s speed *r* (the speed of course`s change),

$$r = \frac{d\psi}{dt}$$

Mathematical model of controlling needs to be simplified. Why some things are assumed: the depth of the sea is infinite, there are no other objects near the ship and the velocity of the ship is constant.

Also, the assumption is that the ship is in a stable condition. So, the variables which represent the dynamic behaviour of the ship are just slightly different from the values in a stationary state. Stationary state of the ship is considered to be the state in which the ship is moving headway along a uniform straight line.[2]

At first glance steering a ship along a course and sailing along a trajectory can be equalized. Namely, in theory, this would be the ideal case, that is, ship manoeuvring without the influence of external disturbances [2].

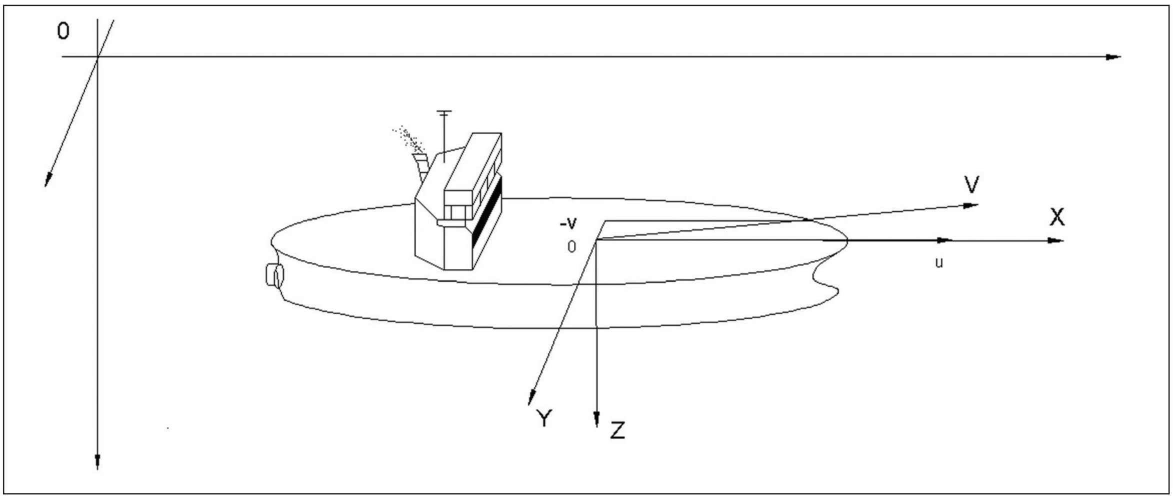

Figure 5 The coordinated system of a ship

Slika 5. Koordinatni sustav broda

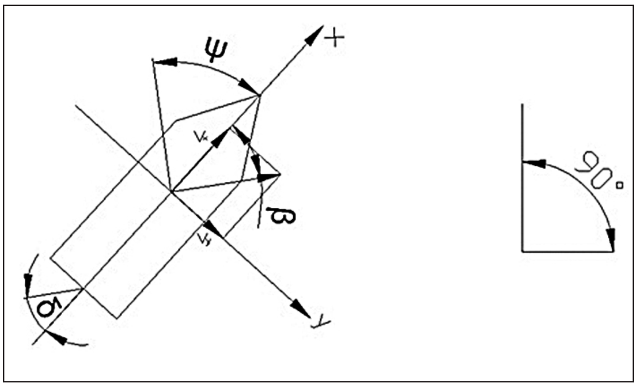

Figure 6 A description of steering a ship along a course in moving coordinated system
Slika 6. Prikaz gibanja broda po kursu u pomičnom koordinatnom sustavu

In the case of a ship's motion, this is practically impossible, therefore it is necessary to create a special system for such purposes. An indispensable part of such a system, which is not present in the classical ships of 20 years ago, is a system for the exact determination of the ship's position in geo reference system. The problem lies in the fact that it is not possible to use the existing navigational systems since the requests for the accurate guidance of a ship along a trajectory allows only for a couple of meters' error. Systems with the necessary accuracy (for example a GPS satellite) are used for such a purpose.

Following the fitting of the entire system, an adequate microprocessor block is installed, which determines the error between the current position of the ship and the given trajectory. The error is then calculated in the adequate steering block into a component of the course error. This course error is processed by the existing autopilot.

2.1.2.1 THE TRAJECTORY ERROR / *Greška trajektorije*

There is one another use of the term trajectory. The trajectory error is considered to be the shortest distance between the current position of the ship and the given trajectory. Mathematically, this is the distance of a point from the straight line since the trajectory can be divided into a finite number of broken-up linear components. [2] [8]

Although there are various ways to calculate the trajectory error, we will name just one that corresponds to the calculation algorithm, i.e. microprocessor.

The basic trigonometric equations are applied to the picture 7.

ψ_t^k - course of the trajectory (kth fragment) (the angle marked with arc)

(x_k, y_k) - final point of the trajectory (kth fragment)

(x_T, y_T)- currently position of the ship

d - trajectory error

D - distance to the kth fragment of the trajectory

From the Fig. 6.there are following equations:

$$\Delta x = x_k - x_T \tag{9}$$

$$\Delta y = y_k - y_T \tag{10}$$

$$a = \Delta y \cos \Psi_t^k \tag{11}$$

$$b = \Delta x \sin \Psi_t^k \tag{12}$$

$$c = \Delta y \sin \Psi_t^k \tag{13}$$

$$e = \Delta x \cos \Psi_t^k \tag{14}$$

$$d = b - a \tag{15}$$

$$D = c + e \tag{16}$$

Merging equations from (9) to (16) we get the equation (17) which describes the trajectory error.

$$d = \Delta x \sin \Psi_t^k - \Delta y \cos \Psi_t^k \tag{17}$$

The distance to the k[th] fragment of the trajectory is given with equation (18).

$$D = \Delta y \sin \Psi_t^k + \Delta x \cos \Psi_t^k \tag{18}$$

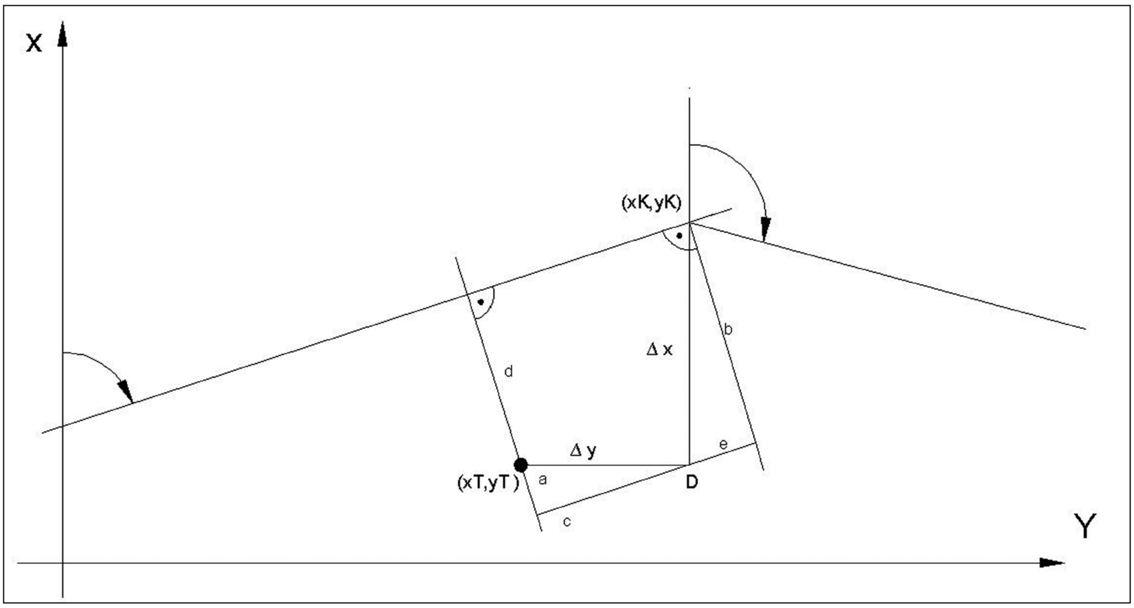

Figure 7 Determination of the trajectory error
Slika 7. Određivanje greške trajektorije

From the equations (9 - 16) the trajectory error *d* is obtained by equation (17). However, we have also obtained the distance to the end of the k^{th} rectilinear part of the trajectory *D* (equation (18)), which is an additional useful piece of information.

It is necessary to note that given equation represents static error of the trajectory of the vessel. It did not consider the dynamics of motion associated with the concept of controllability and maneuverability of the vessel.

2.1.2.2 MATHEMATICAL MODEL OF STEERING A SHIP ALONG THE GIVEN TRAJECTORY / *Matematički model upravljanja brodom duž dane trajektorije*

Mathematical model steering a ship along a given trajectory is, in fact, mathematical model moving a ship along a given course extended with kinematic relation. In the realistic situation there is one more dynamic relation: the rudder - sideways yawing. But, this component can be unattended because of simplifying the steering a ship along a trajectory.

Expressions for mathematical model are derived from the fig. 8.

In the figure 8. there are:

ψ_t - course of the trajectory

$-\Delta\psi$ - difference of trajectory`s course

U - speed of ship`s progress

d - trajectory error

$\dot{d}$ - alternation of trajectory error

The expression for the alternation of trajectory error (from the Fig. 8.)

$$\dot{d} = U\sin(-\Delta\psi) \quad (19)$$

Where U is the speed of ship`s progress.

$$\Delta\psi = \psi_t - \psi \quad (20)$$

$\Delta\psi$ is course`s error while it is assumed that ordered course is equal to the trajectory`s course, which is the most common case. [18]

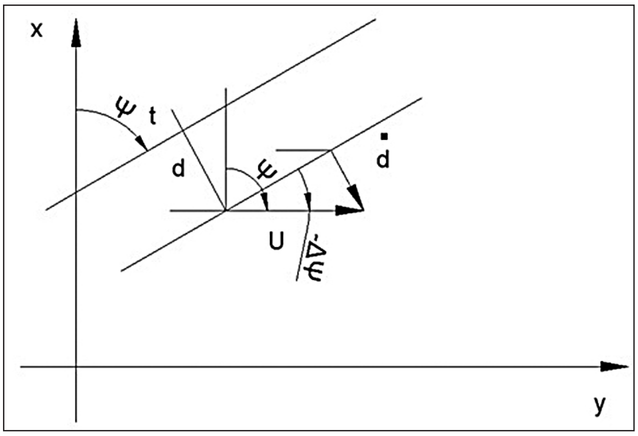

Figure 8 Kinematic bracing steering a ship along a trajectory
Slika 8. Kinematska sprega vožnje po trajektoriji

The small-angle approximation is a useful simplification of the basic trigonometric functions which is approximately true in the limit where the angle is small.

For small amounts of course`s error when $\Delta\psi < 15^0$ it can be written:

$$\dot{d} = -U\Delta\psi \quad (21)$$

Relevant kinematic relation can be included in present mathematical model of steering a ship along a course. The following block scheme is given for the explanation.

The transfer function G_0 represents the transfer function of open circle riding a ship along a trajectory (fig. 8.) which is composed from ship`s autopilot, the rudder`s device and the ship`s model of the rudder – a course. [18]

ψ_t - course of the trajectory

U - speed of ship`s progress

d - trajectory error

The trajectory error is treated and transformed in the form of control signal which is added to the course`s error. That situation is represented in Fig.10. in the form of control signal $\Delta\overline{\psi}$. [18]

The transfer function of driving along a trajectory has a form:

$$G_t = \frac{d(p)}{\Delta\overline{\psi}(p)} = \frac{-U}{p}\cdot G(p) \quad (22)$$

Where *G*(*p*) is the transfer function of automatic controlling along a course:

$$G_p = \frac{G_0(p)}{1+G_0(p)} \quad (23)$$

Almost all modern (recent) ship`s autopilots have adaptive ability. Therefore, the transfer function can be considered as the constant transfer function of a closed circle (there are no impacts of parameters of the drive as the speed of a ship, the condition of the cargo, the external disturbances etc.) [18]

In that case, the transfer function of a closed system driving a ship along a trajectory can be represented in a final form which is used for designing systems of driving a ship along a given trajectory.

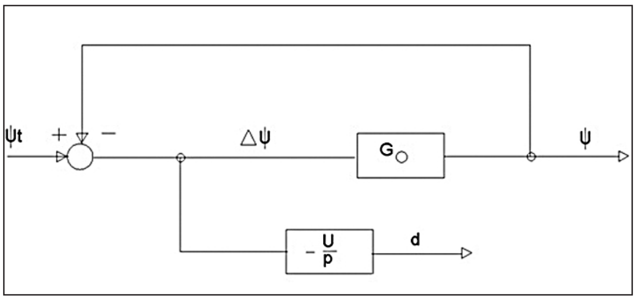

Figure 9 The transfer function of an open circle of riding a ship along a trajectory
Slika 9. Prijenosna funkcija otvorenoga kruga vožnje po trajektoriji

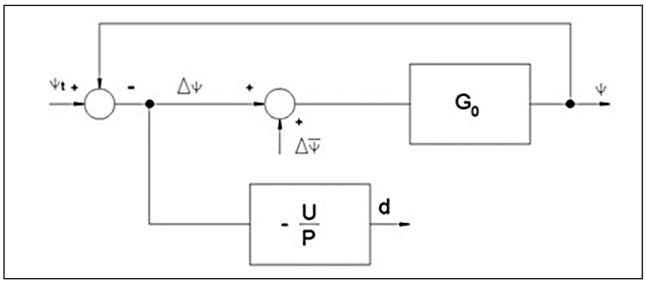

Figure 10 The transfer function of open circle driving the ship along a given trajectory (with absorbed trajectory error)
Slika 10. Prijenosna funkcija otvorenoga kruga vožnje po trajektoriji (uz uračunatu grešku trajektorije)

$$G_t(p) = \frac{d(p)}{\Delta\overline{\psi}(p)} = \frac{-U}{p\left(\frac{1}{\omega_0^2}p^2 + \frac{2\xi}{\omega_0}p + 1\right)} \tag{24}$$

Values are:

G_t - transfer function of driving along a trajectory

d - trajectory error

$\Delta\psi$ - difference of trajectory`s course

U - speed of ship`s progress

ω_0 - own frequency of closed system

ξ - camber (overshoot)

The accent is on the fact that equation (24) can be used in practice. Namely, for particular ship values of ω_0 and ξ are constant and the value of speed of ship`s progress U can be measured.

So, using adaptive autopilot and adequate systems of steering a ship along a given trajectory, a specific ship will have adaptive ability with minimal demands of trajectory error compensation.

3. CONCLUSION / *Zaključak*

Scientific discoveries in different fields of science have served scientists in other fields as the basis for their research. Among all of those discoveries, new findings and theorems, those in the field of mathematics are especially outstanding. It is said that „mathematics is the queen and servant of all the other sciences". Also, the progress in computers has created the possibility of arriving quickly at the calculation of problems that had been previously either almost impossible or actually impossible to solve.

The development of new technologies related to the exploitation of the underwater world, the use of the sea and the underwater world for traffic purposes, as well as the progress of mathematics, have led to a new way of steering a ship - sailing along a given trajectory. At first glance, steering a ship along a course and sailing along a trajectory can be equalized. Namely, in theory, this would be the ideal case, that is, ship manoeuvring without the influence of external disturbances. Mathematical model steering a ship along a given trajectory is, in fact, mathematical model moving a ship along a given course extended with kinematic relation. An adequate microprocessor block is installed, which determines the error between the current position of the ship and the given trajectory. The error is then calculated in the adequate steering block into a component of the course error. This course error is processed by the existing autopilot. The trajectory error is treated and transformed in the form of control signal which is added to the course`s error.

Almost all modern (recent) ship`s autopilots have adaptive ability. Therefore, the transfer function can be considered as a constant transfer function of closed circle. In that case, the transfer function of a closed system driving a ship along a trajectory can be presented in final form which is used for designing systems of driving a ship along a given trajectory. So, using adaptive autopilot and adequate systems of steering a ship along a given trajectory, a specific ship will have adaptive ability with minimal demands of trajectory error compensation.

This also allows for the safer navigation of many ships, which today, along with the rough sea and strong sea currents that follow them from the first days of sailing, also face many other problems (wars, pirates).

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